Introduction to Operational Modelling

Flávio Martins
fmartins@ualg.pt

European University of the Seas
SEA EU
UAlg CIMA
UNIVERSIDADE DO ALGARVE
CENTRO DE INVESTIGAÇÃO MARINHA E AMBIENTAL

Summary

- What is a Model?
- Transport of a property; Eulerian vs Lagrangian
- Discretization in space and in time
- Model limitations
- Initial and boundary conditions
- Downscaling
- Types of models
- Operational modelling cycle
- Data assimilation, analysis and reanalysis
- Model Calibration and Validation

What is a model?

What is a model?

What is a model?

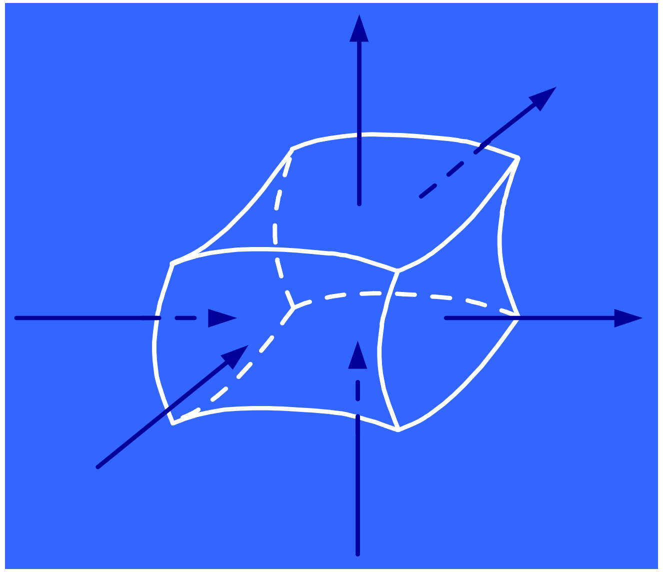
What is a model?

What is a model?

- Again: What is a model?
  - Abstracted representation of complex, ‘real world’ System (i.e. abstraction or simplification of reality)
- Why modelling? $M(u) = y$

  u: Data inputs; M: model representation; y: resulted outputs
  - Given M and u find y: Simulation, Prediction, Understanding;
  - Given u and y find M: System Identification;
  - Given M and y find u: Inverse problem, Management, Decision Making

Again: What is a model?

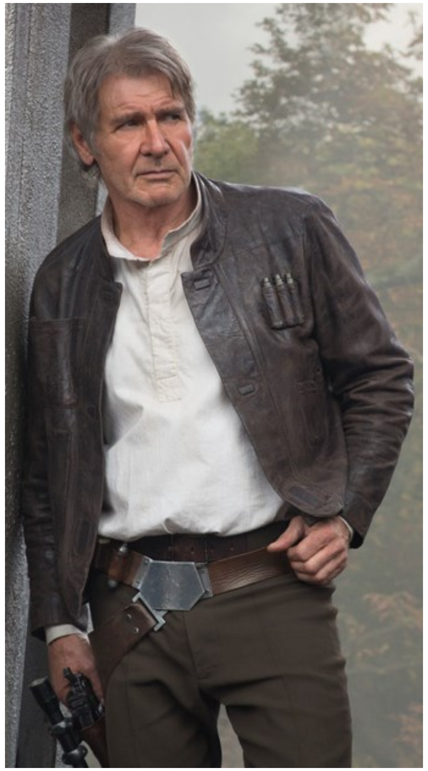


$$\frac{\partial}{\partial t}\iiint_{CV}\beta dV = -\iint_{surface}\left(\beta\vec{v}.\vec{n} - A\left(\vec{\nabla}\beta\right)\cdot\vec{n}\right)dA +$$

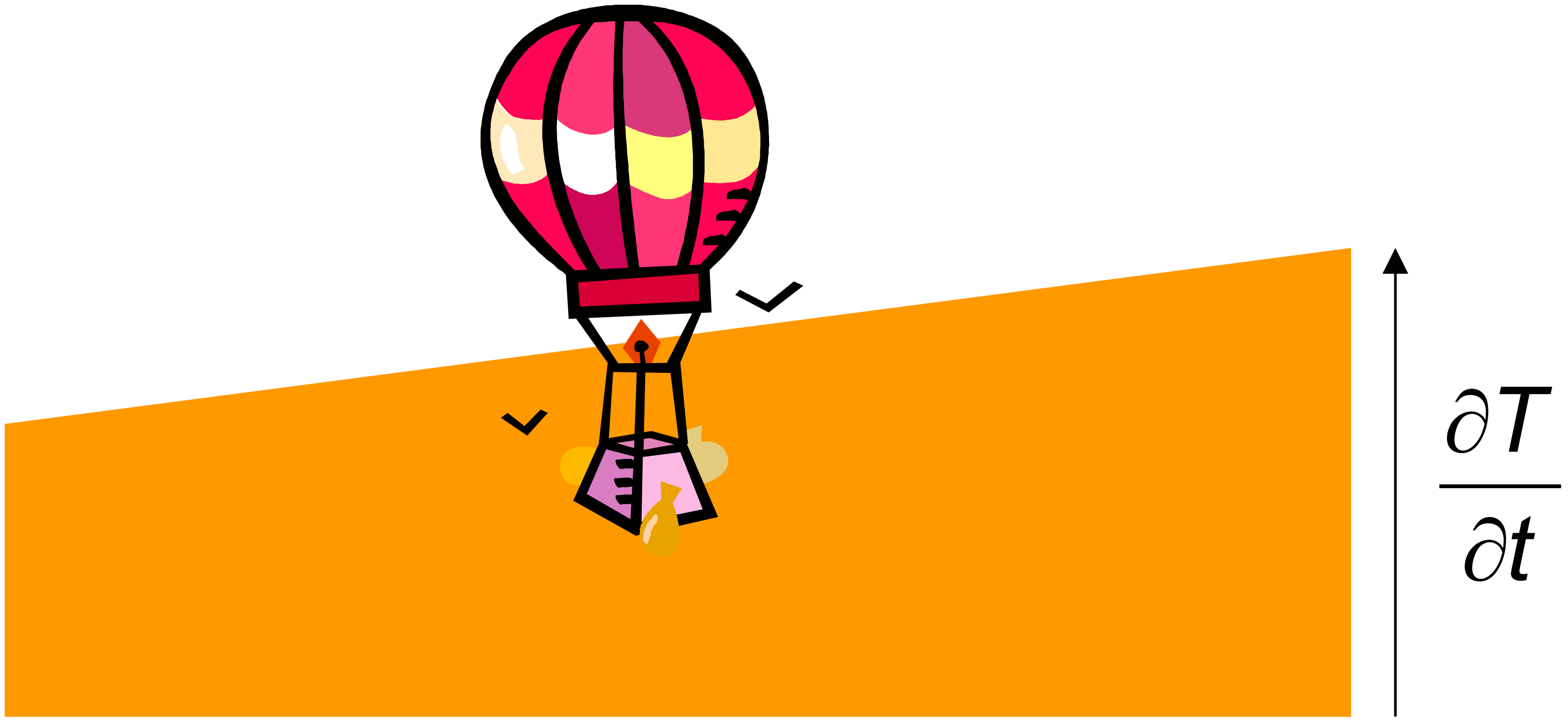
$$+(S_o - S_i)$$

Unlike other models.....

Mathematical models improve with age.....

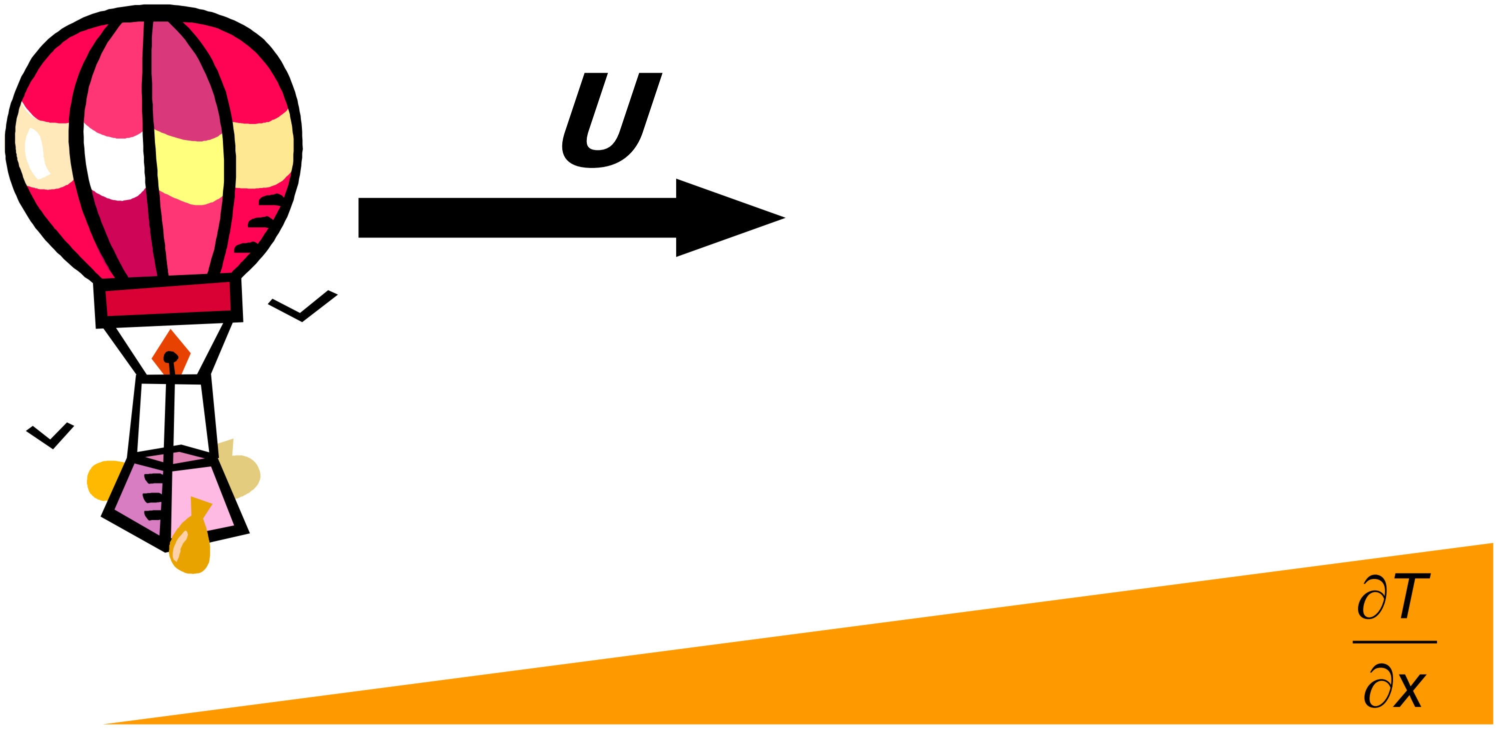


Transport of a property



$$\frac{DT}{Dt} = \frac{\partial T}{\partial t}$$

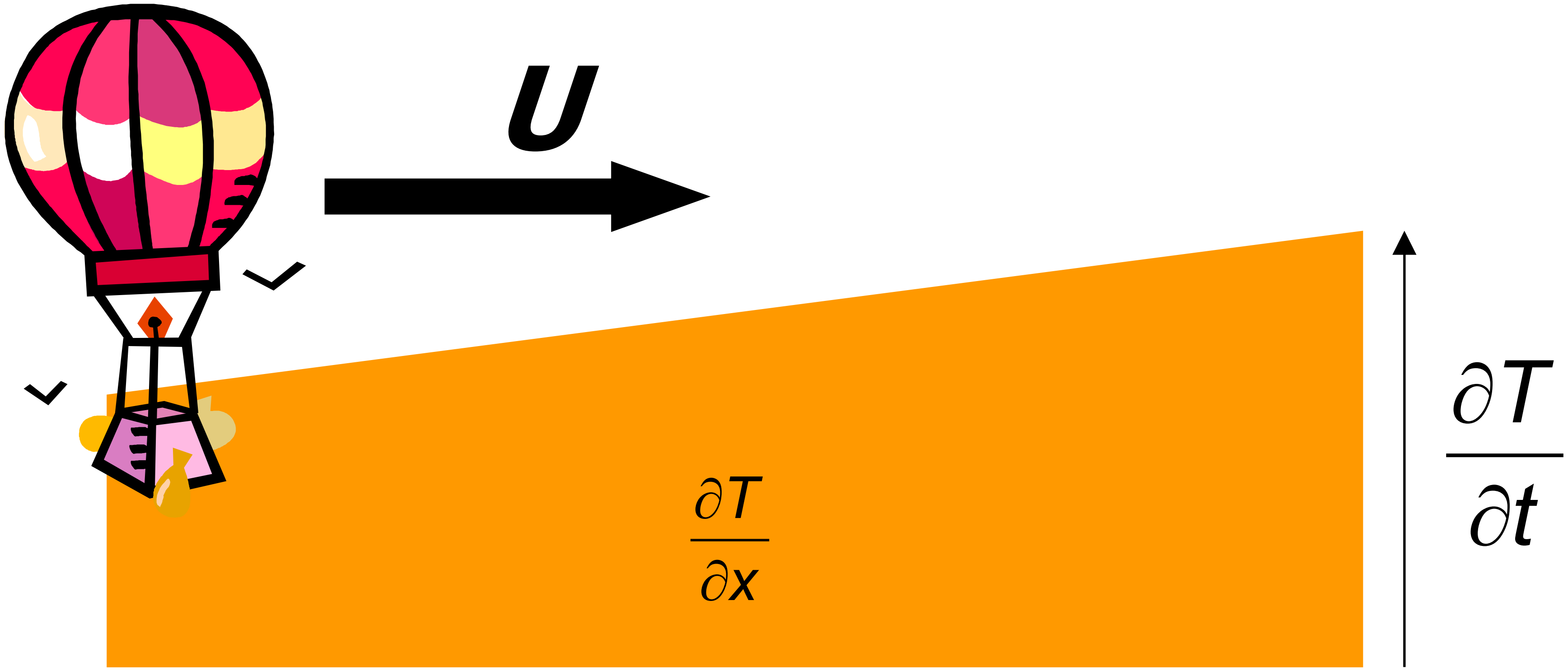
Transport of a property



$$\frac{\partial T}{\partial t} = 0$$

$$\frac{DT}{Dt} = U\frac{\partial T}{\partial x}$$

Transport of a property

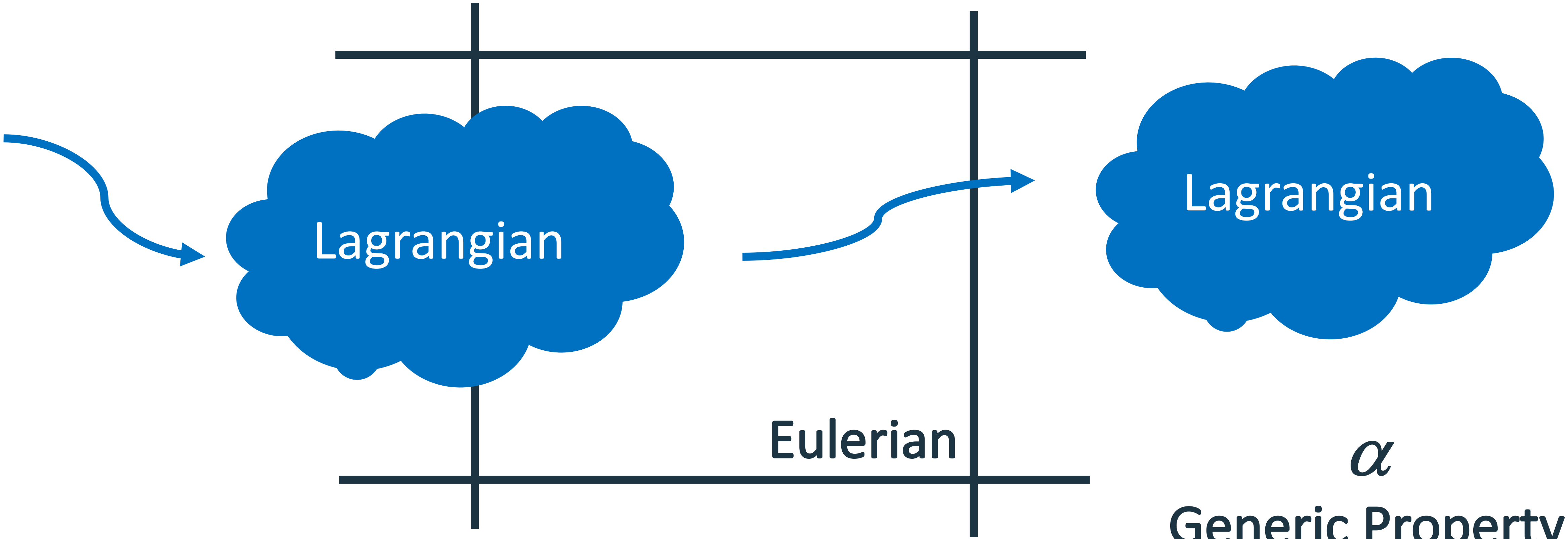


$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x}$$

Temporal Variation

Advective Variation

Lagrangian vs Eulerian approach



α

Generic Property

$$\frac{D\alpha}{Dt} = \frac{\partial \alpha}{\partial t} + U\frac{\partial \alpha}{\partial x}$$

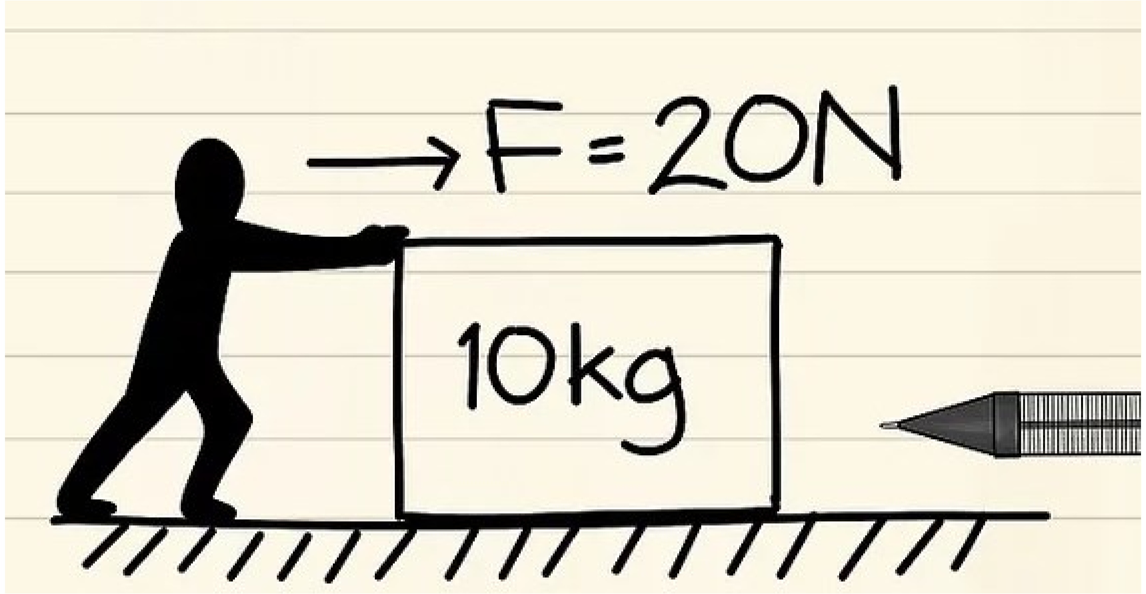
α = v Acceleration

Discretization in space and in time

Which Equations?

Newton's Law for a solid:

$$\vec{F} = m\vec{a}$$

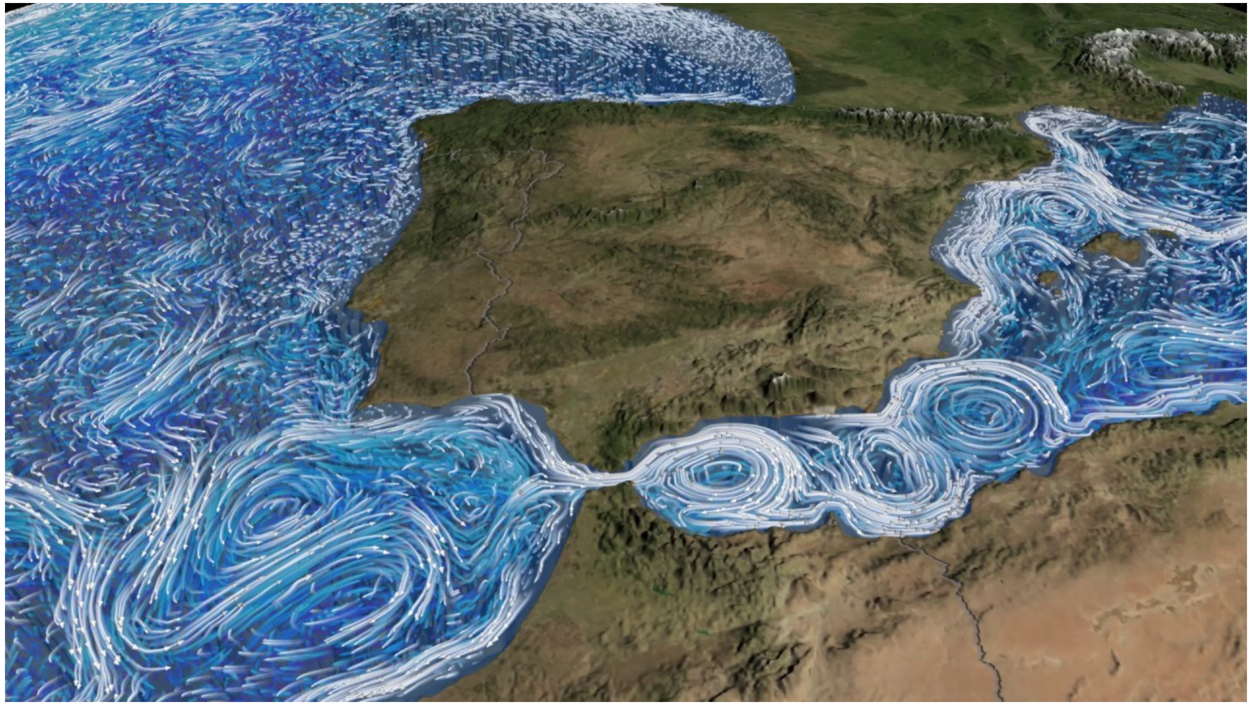


Discretization in space and in time

Which Equations?

Newton's Law for a fluid:

$$\overrightarrow{F(x,y,z,t)} = m\,\overrightarrow{a(x,y,z,t)}$$



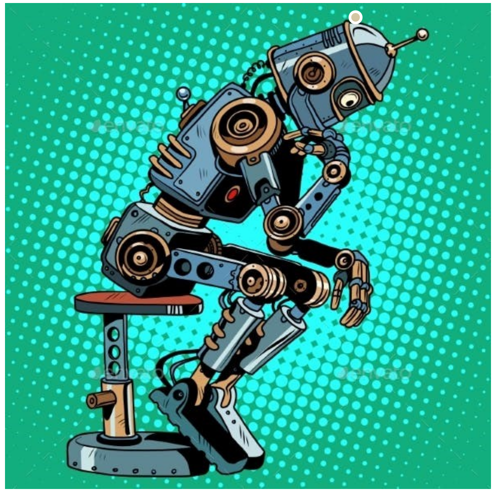
Field equations

Discretization in space and in time

Which Equations?

Newton's Law for a fluid = Navier Stokes Equations

1,000,000 $
Prize !



$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0 \qquad \frac{\partial p}{\partial x_3} = -\rho g$$

$$\frac{\partial u_1}{\partial t} + \frac{\partial (u_j u_1)}{\partial x_j} = f u_2 - g \frac{\rho_n}{\rho_0} \frac{\partial \eta}{\partial x_1} - \frac{1}{\rho_0} \frac{\partial p_s}{\partial x_1} - \frac{g}{\rho_0} \int_z^{\eta} \frac{\partial \rho'}{\partial x_1} dx_3 + \frac{\partial}{\partial x_j}\left(A_j \frac{\partial u_1}{\partial x_j} \right)$$

$$\frac{\partial u_2}{\partial t} + \frac{\partial (u_j u_2)}{\partial x_j} = -f u_1 - g \frac{\rho_n}{\rho_0} \frac{\partial \eta}{\partial x_2} - \frac{1}{\rho_0} \frac{\partial p_s}{\partial x_2} - \frac{g}{\rho_0} \int_z^{\eta} \frac{\partial \rho'}{\partial x_2} dx_3 + \frac{\partial}{\partial x_j}\left(A_j \frac{\partial u_2}{\partial x_j} \right)$$

Discretization in space and in time

$$\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}=0 \qquad \frac{\partial p}{\partial x_3}=-\rho g$$

$$\frac{\partial u_1}{\partial t}+\frac{\partial (u_j u_1)}{\partial x_j}=fu_2-g\frac{\rho_n}{\rho_0}\frac{\partial \eta}{\partial x_1}-\frac{1}{\rho_0}\frac{\partial p_s}{\partial x_1}-\frac{g}{\rho_0}\int_z^{\eta}\frac{\partial \rho'}{\partial x_1}dx_3+\frac{\partial}{\partial x_j}\left(A_j\frac{\partial u_1}{\partial x_j}\right)$$

$$\frac{\partial u_2}{\partial t}+\frac{\partial (u_j u_2)}{\partial x_j}=-fu_1-g\frac{\rho_n}{\rho_0}\frac{\partial \eta}{\partial x_2}-\frac{1}{\rho_0}\frac{\partial p_s}{\partial x_2}-\frac{g}{\rho_0}\int_z^{\eta}\frac{\partial \rho'}{\partial x_2}dx_3+\frac{\partial}{\partial x_j}\left(A_j\frac{\partial u_2}{\partial x_j}\right)$$

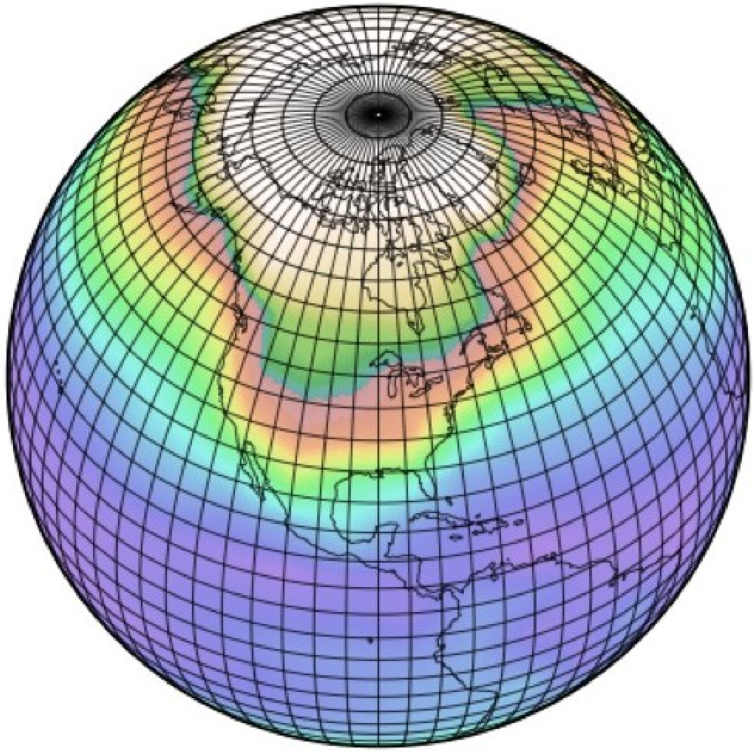
Discretization

$$\begin{cases} a_1v_1+a_2v_2+a_3v_3+\cdots \\ b_1v_1+b_2v_2+b_3v_3+\cdots \\ c_1v_1+c_2v_2+c_3v_3+\cdots \\ \cdots \\ \cdots \\ Millions \end{cases}$$

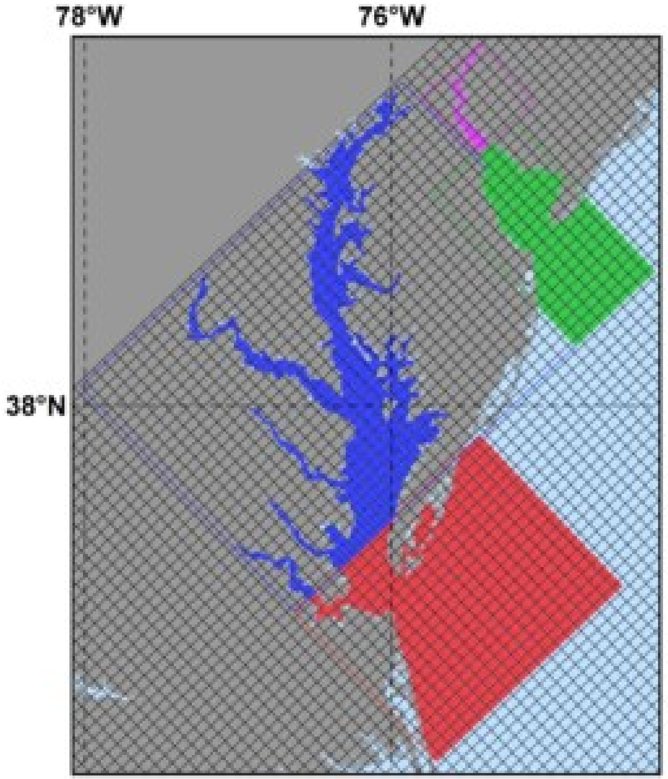
Solving

Discretization in space and in time

Discretization in space (mesh type)



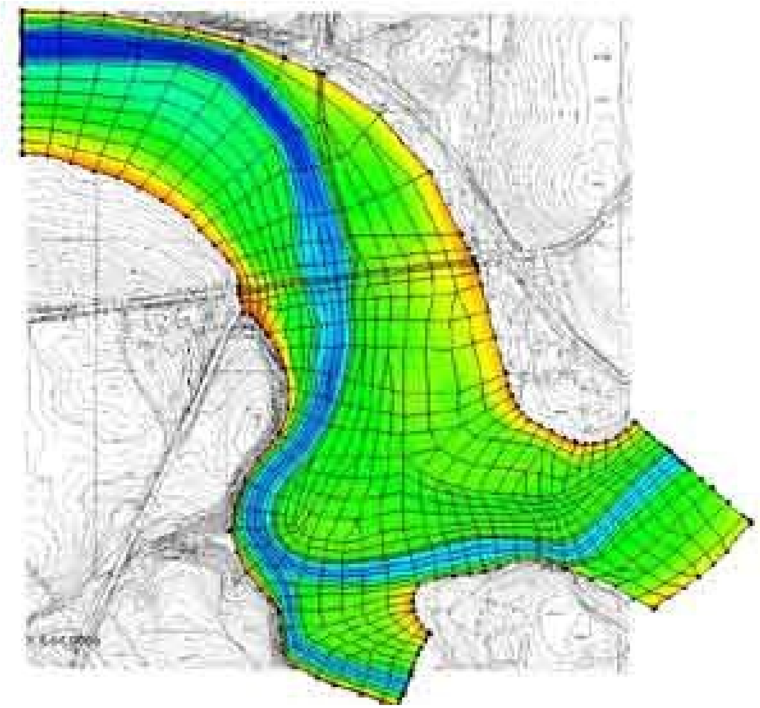
Geographic (Cartesian (ϕ, θ))



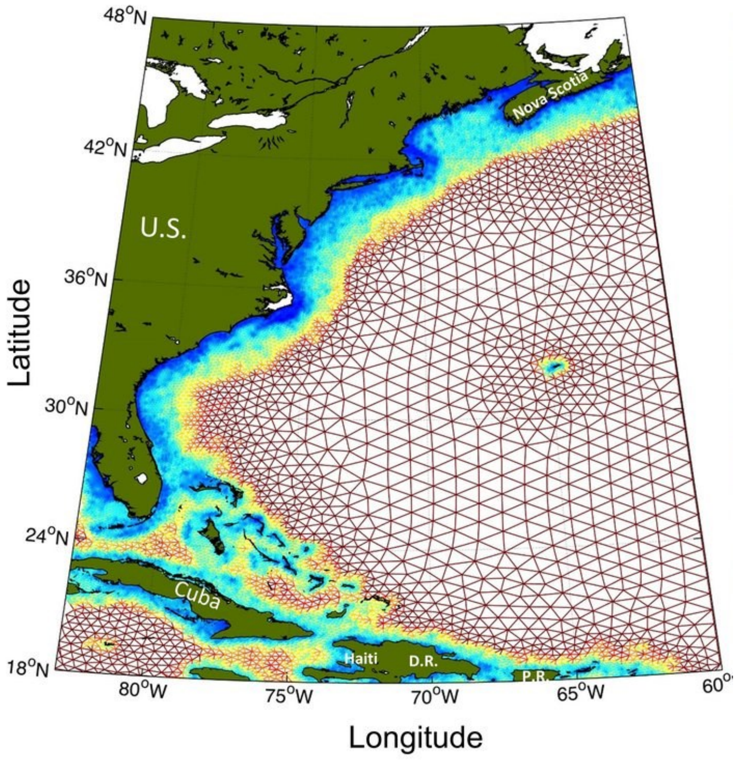
Cartesian (x, y)

Discretization in space and in time

Discretization in space (mesh type)



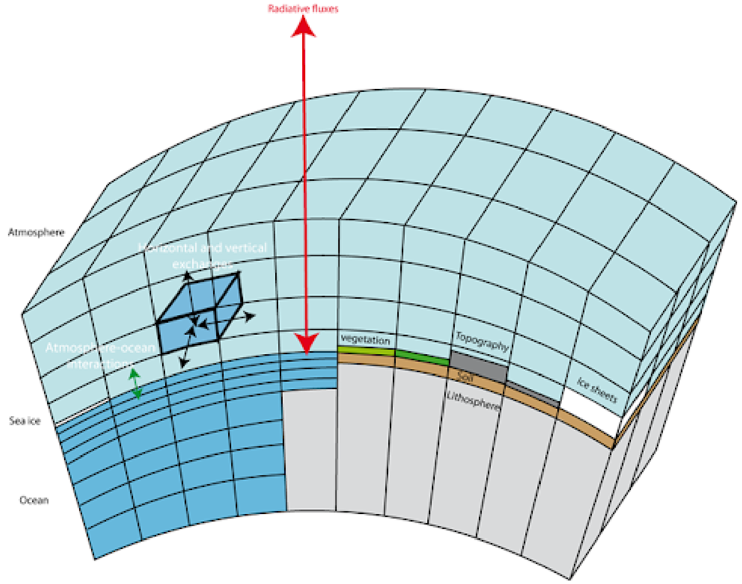
Curvilinear Orthogonal



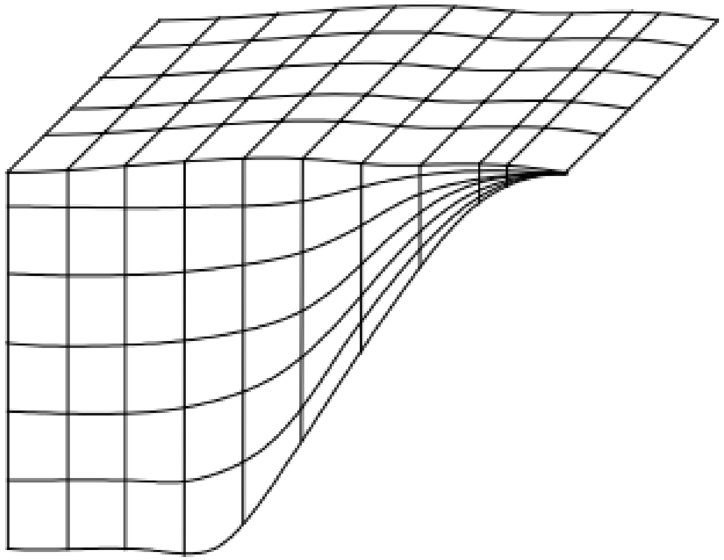
Triangular (unstructured)

Discretization in space and in time

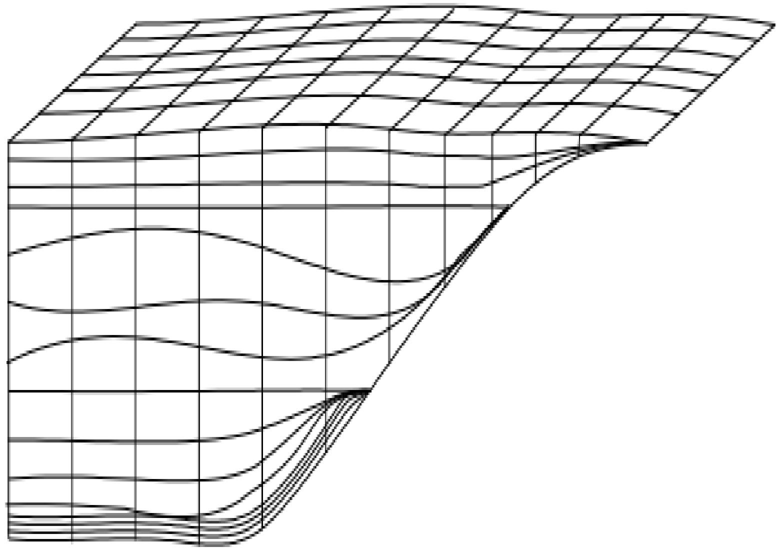
Discretization in space (vertical)



Cartesian

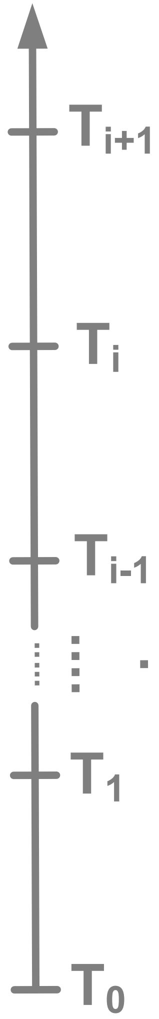


Sigma
(Terrain Following)



Generic

Discretization in space and in time



Discretization in time

- Explicit methods: $u^{i+1} = f(u^i)$
- Implicit methods: $u^{i+1} = f(u^{i+1})$
- Semi- implicit methods: $u^{i+1} = f(u^{i+1}, u^i)$

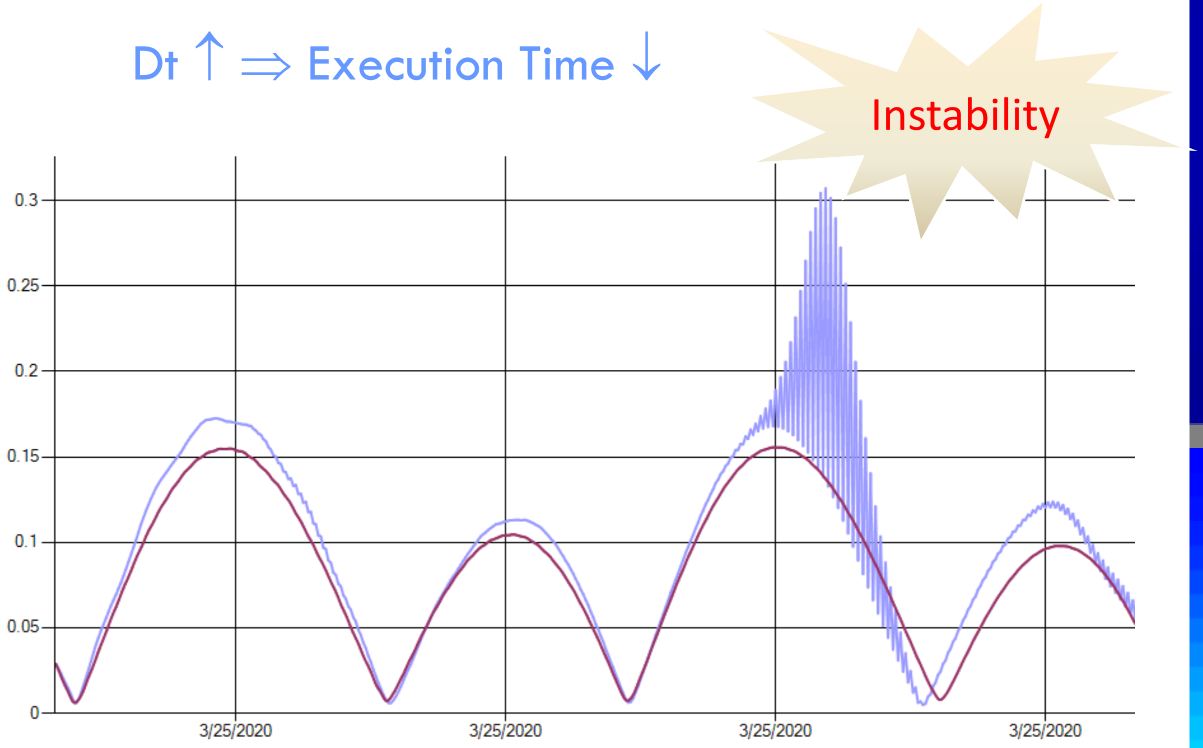
e.g.: ADI

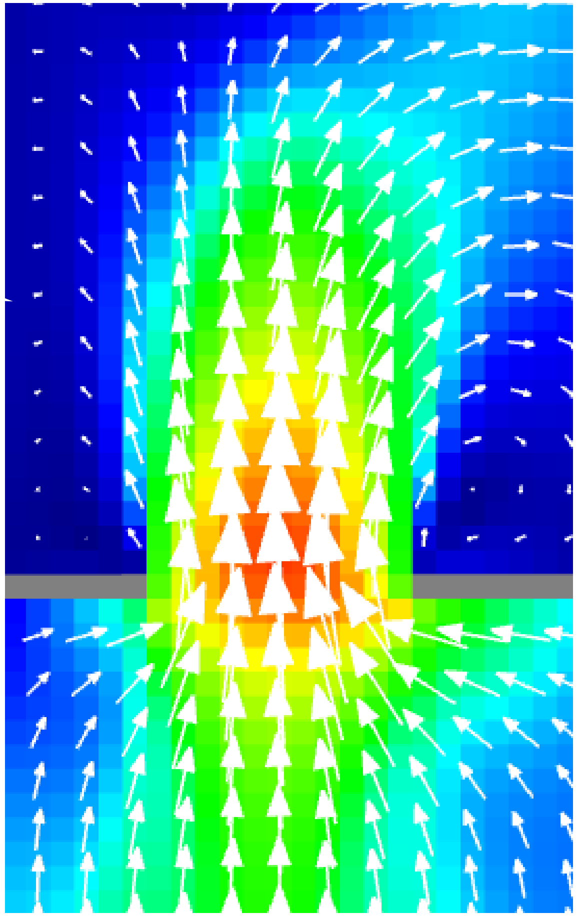
Model limitations

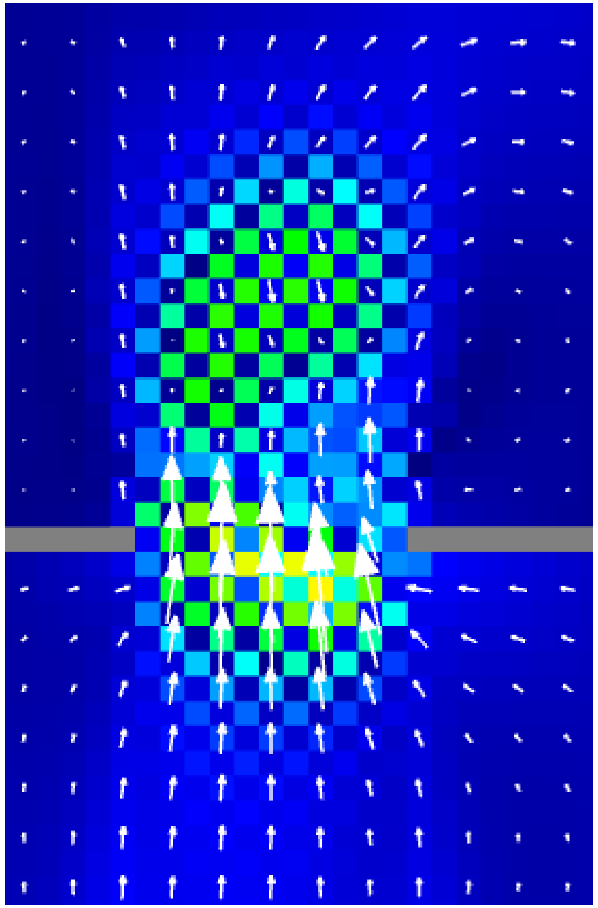
$$Courant = \frac{v\,\Delta t}{\Delta x}$$

Dt ↓ ⇒ Execution Time ↑

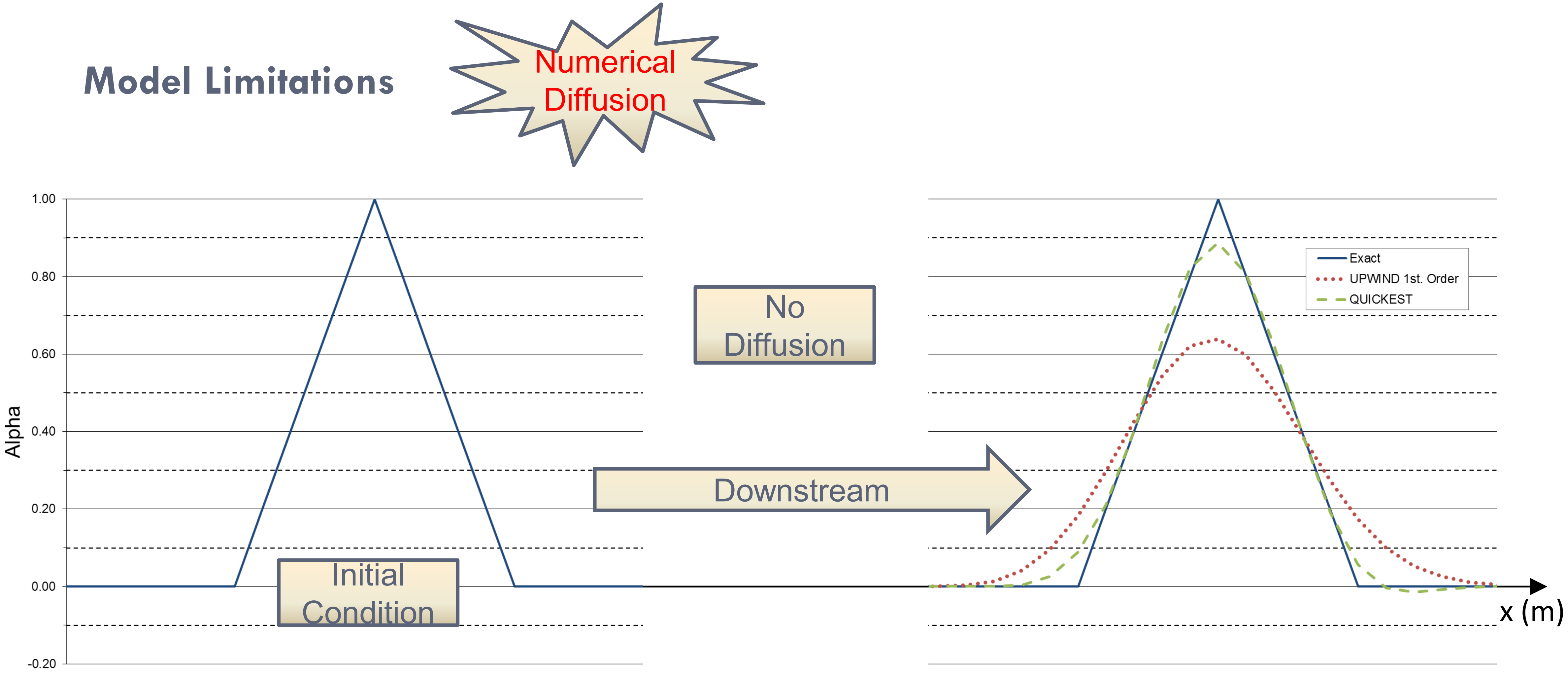
Dt ↑ ⇒ Execution Time ↓







Model limitations



Model limitations

Model Limitations

Numerical Diffusion

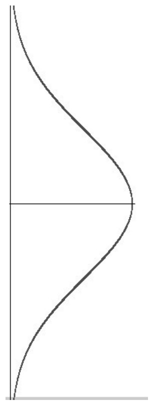
$$Cr = \frac{v\,\Delta t}{\Delta x} = 1.0$$

v

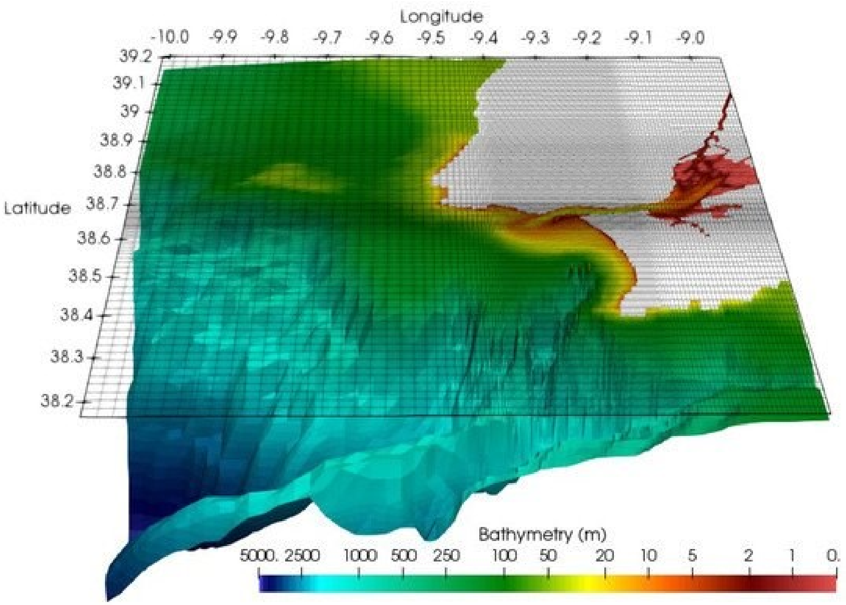
T=0	1	2	3	4
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

$$Cr = \frac{v\,\Delta t}{\Delta x} = 0.5$$

T=0	1	2	3	4
1	0.5	0.25	0.125	0.0625
0	0.5 / 1	1	0.375	0.25
0	0	0.25	0.375 / 1	1
0	0	0	0.125	0.25
0	0	0	0	0.0625



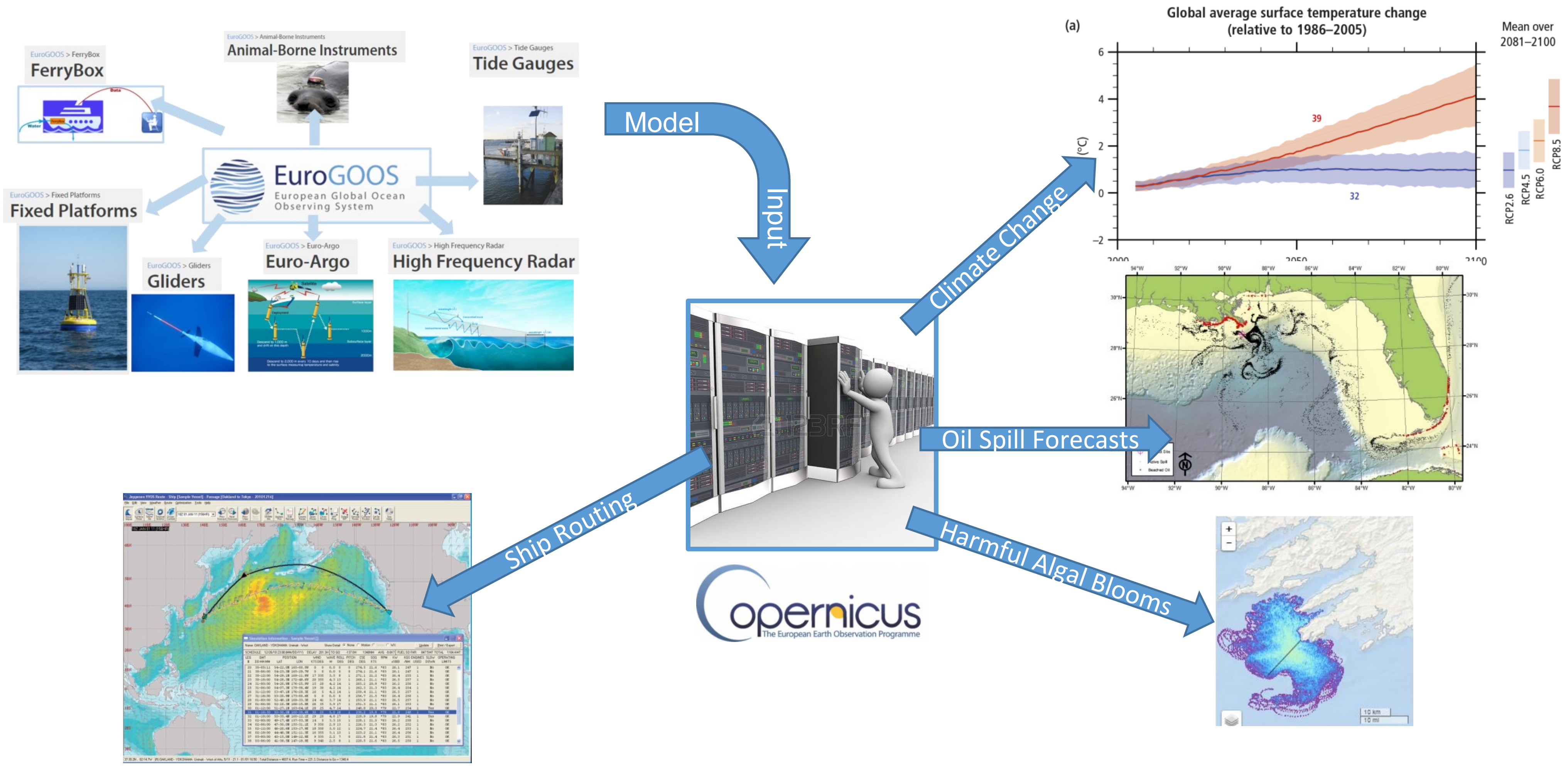
Initial and boundary conditions



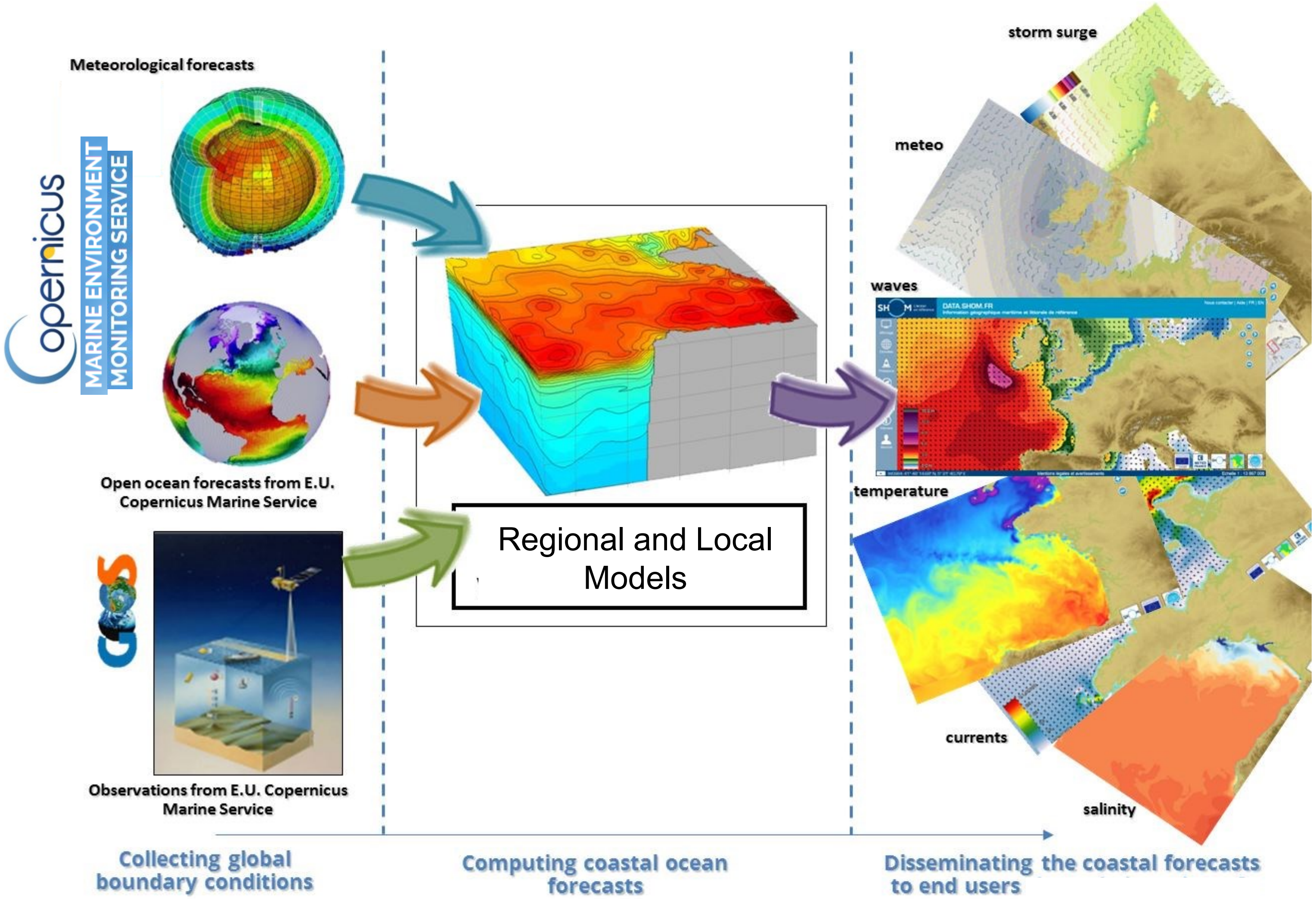
- Initial conditions:
  - Values in every cell
  - Null (very dynamic Sys.)
  - Interpolation
  - Model “spinup”
  - Data Assimilation

- Boundary conditions:
  - Every time step
  - Surface; Bottom; Lateral
  - Spounge Layers
  - Flux Relaxation
  - Radiation Conditions

Downscalling

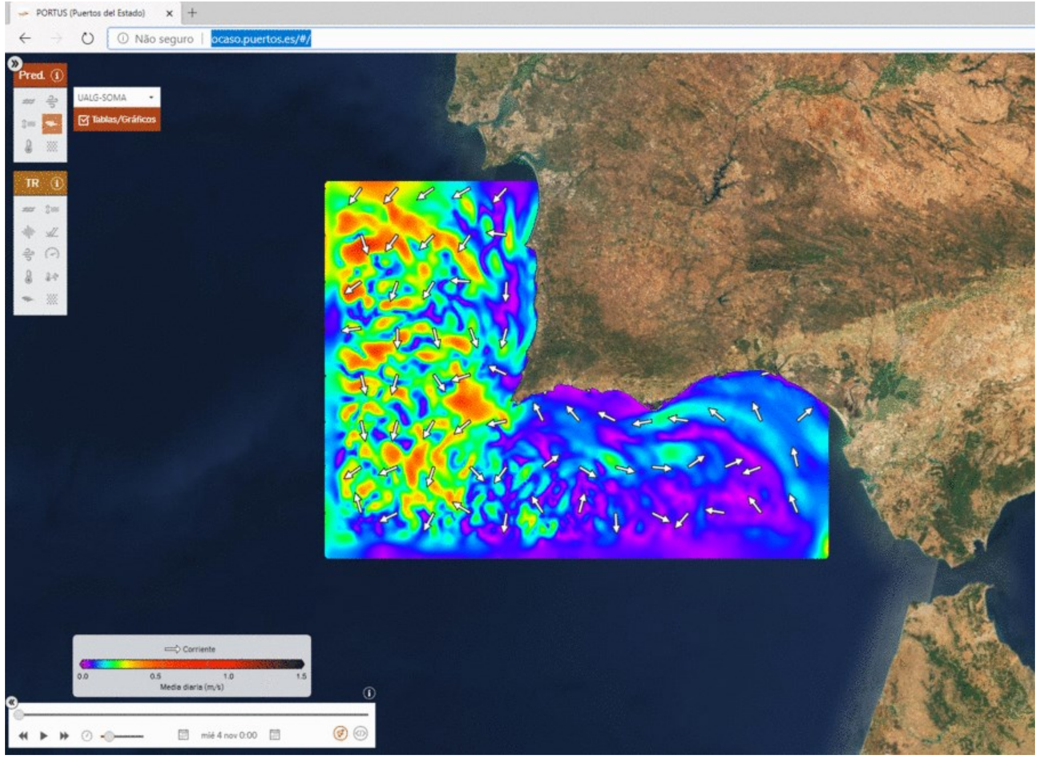


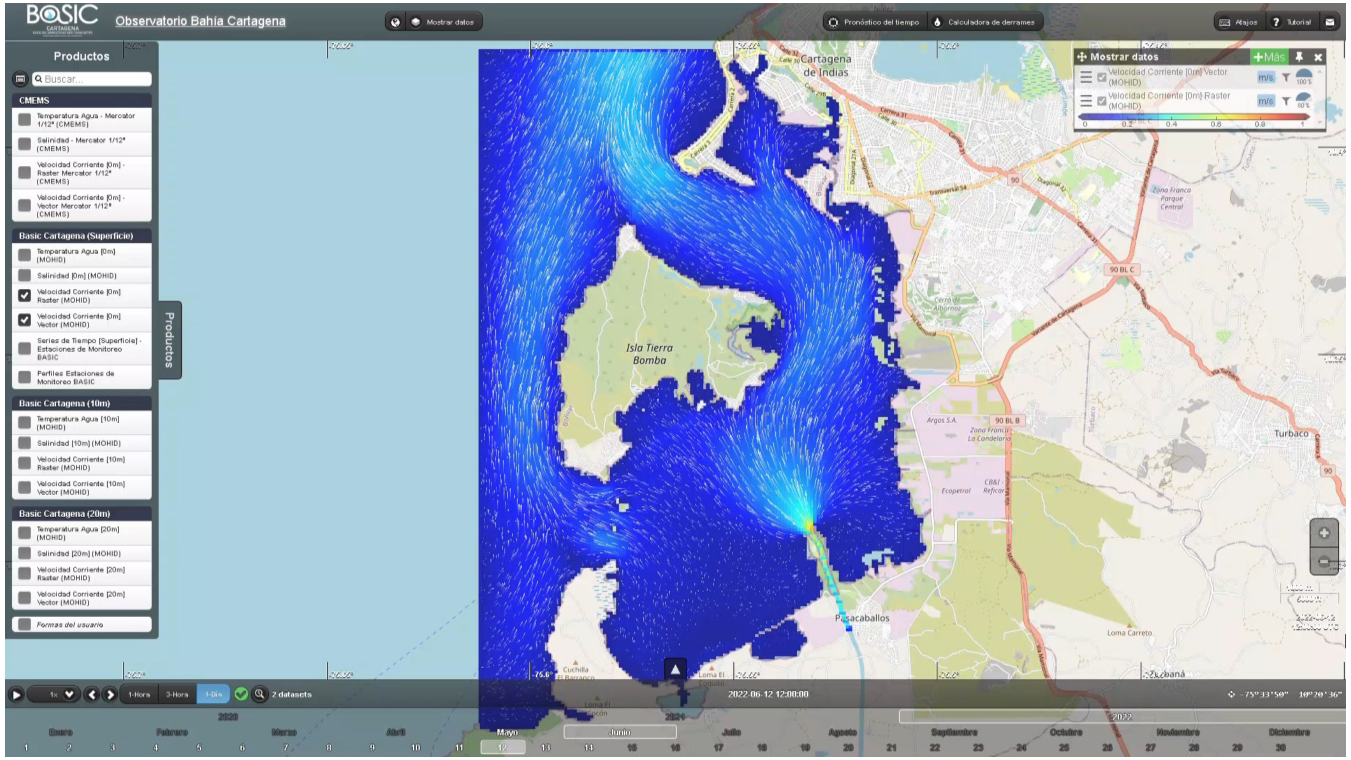
Downscalling



Adapted from www.shom.fr

Downscalling





- SOMA Model (OCASO project)

https://ocaso.puertos.es/#/

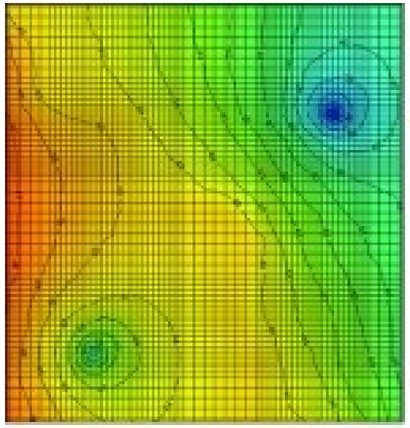
- Cartagena Bay (Colombia) (BASIC project)

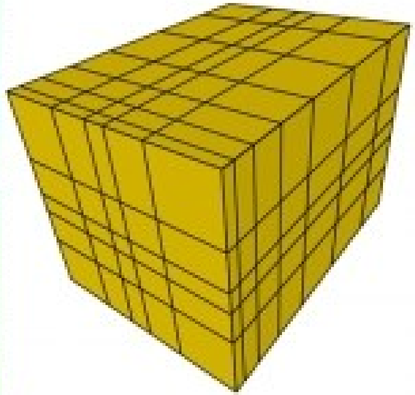
http://bahiacartagena.omega.eafit.edu.co/

Types of models

Finite differences

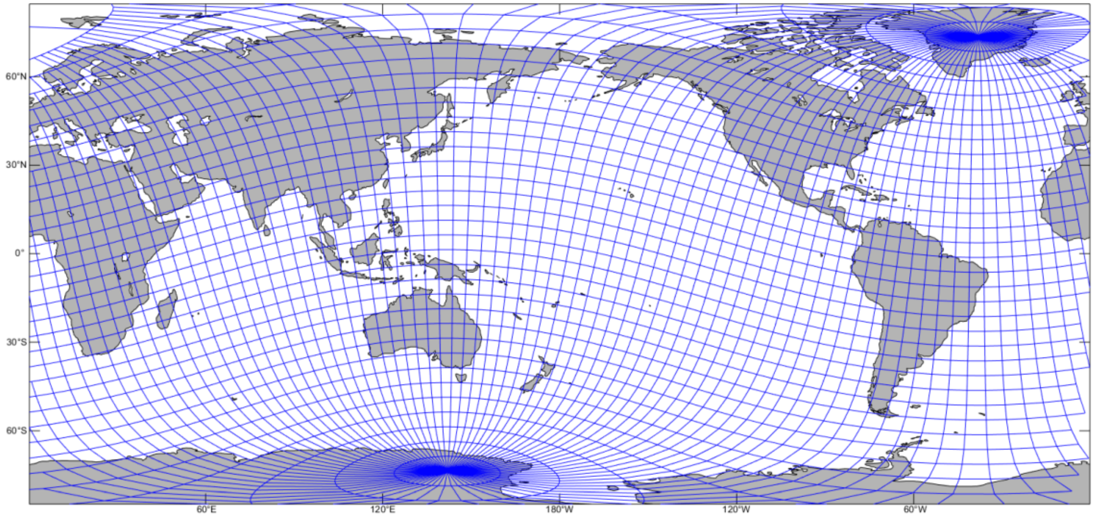
- *Differential Eq. → Algebraic differences equations*
- *Conservative Problems*
- *Curvilinear mesh trough coordinate transformations*
- *Easy for teaching*





Finite Volumes

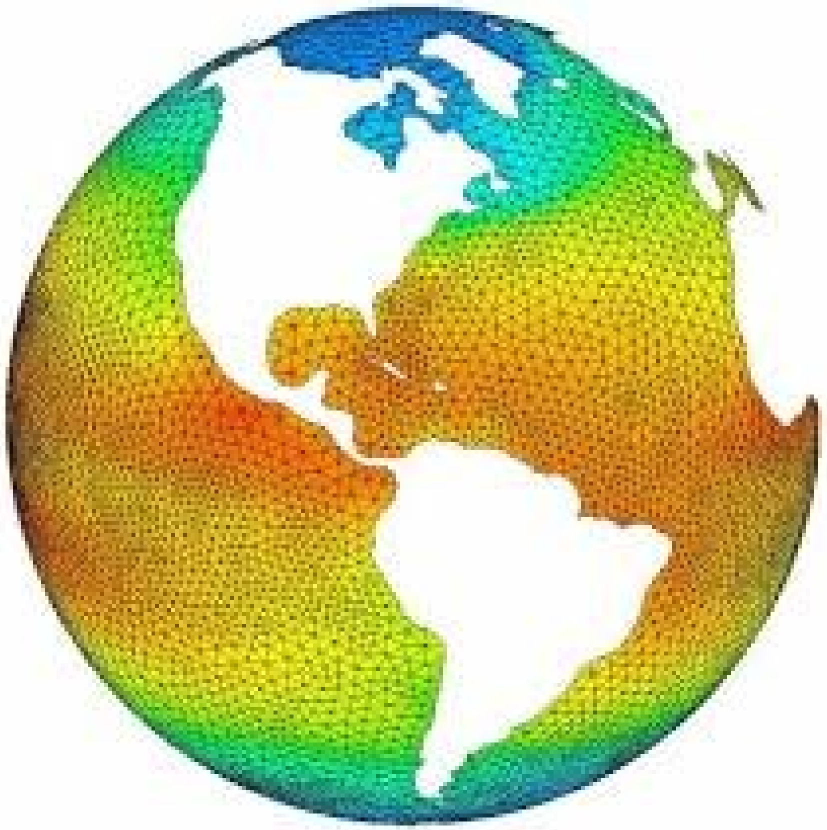
- *CV equations. → Algebraic equations*
- *Conservative by nature*
- *Curvilinear structured and unstructured mesh implemented in the "real" domain.*



Types of models

Finite Elements

- *Differential Eq. → Form functions adjusted trough the minimization of weights.*
- *Conservative Problems*
- *Difficult to include convective terms*
- *Easy to implement unstructured meshes.*



Types of models

- Transport of any property

$$\frac{\partial \alpha}{\partial t} + u_j \frac{\partial \alpha}{\partial x_j} = \frac{\partial}{\partial x_j}\left(K \frac{\partial \alpha}{\partial x_j}\right) + Sources - Sinks$$

- Water Quality (primary production) model

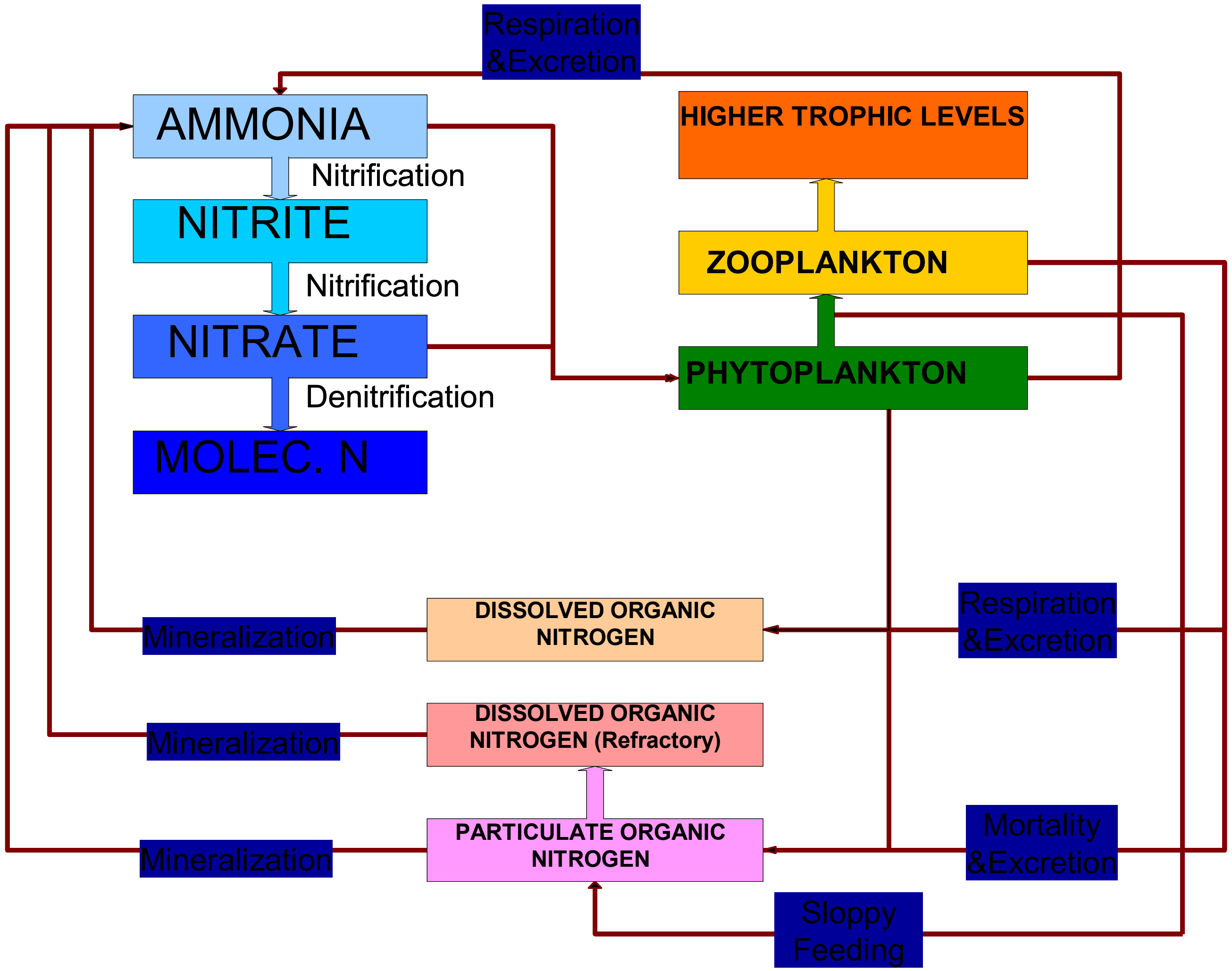
$\alpha =$ Phyto, Zoo, Ammonia,Nitrate, Nitrite, Org.Nitr.,etc.

Sources & Sinks:

Transformation Processes, e.g.: nitrification, grazing, mineralization, denitrification, etc..

Types of models

- Water Quality (primary production) model



Types of models

- Transport of any property

$$\frac{\partial \alpha}{\partial t} + u_j \frac{\partial \alpha}{\partial x_j} = \frac{\partial}{\partial x_j}\left(K \frac{\partial \alpha}{\partial x_j}\right) + Sources - Sinks$$

- Sediment Transport model

$\alpha =$ Sediment Concentration

Sources & Sinks:

Erosion, Resuspension, Sinking, Deposition, Flocculation, Compactation, etc..

Types of models

- Transport of any property

$$\frac{\partial \alpha}{\partial t} + u_j \frac{\partial \alpha}{\partial x_j} = \frac{\partial}{\partial x_j}\left(K \frac{\partial \alpha}{\partial x_j}\right) + Sources - Sinks$$

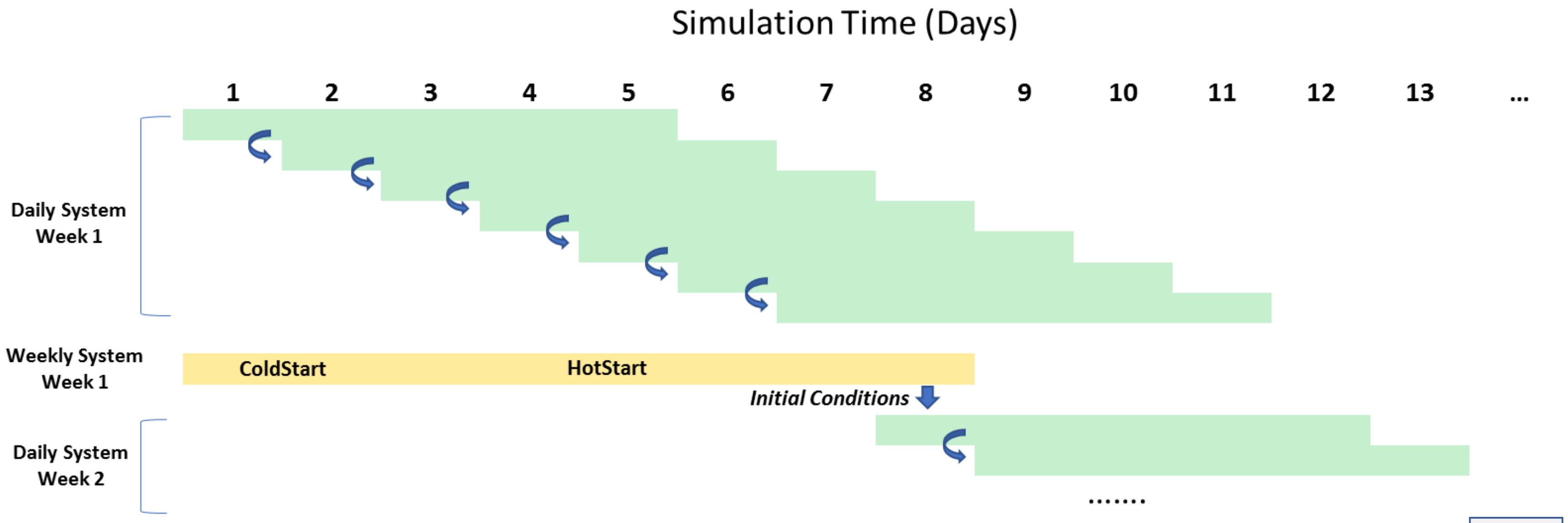
- Water Temperature (thermodynamics)

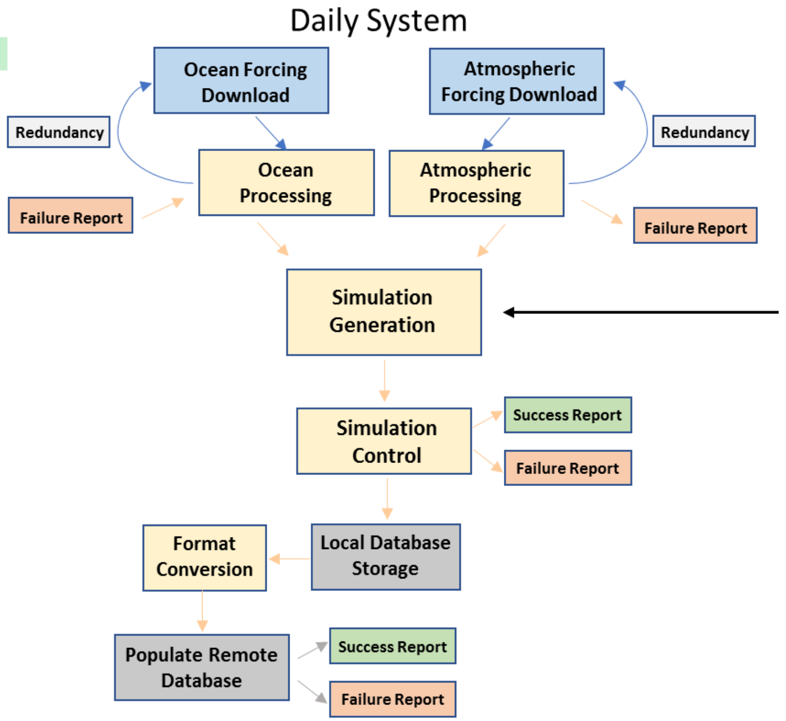
$\alpha = \text{Temperature}$

Sources & Sinks:

Solar radiation; Sensible Heat Loss; Latent Heat Loss; Light Attenuation; Stratification , etc..

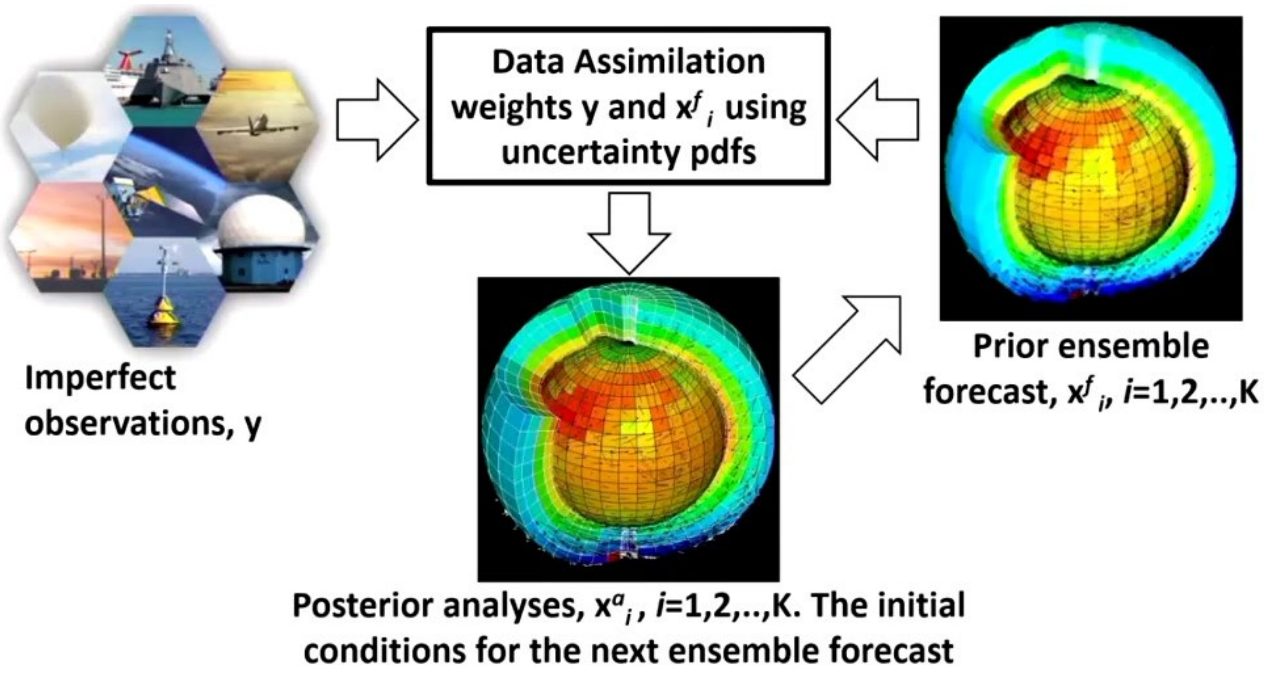
Operational modelling cycle

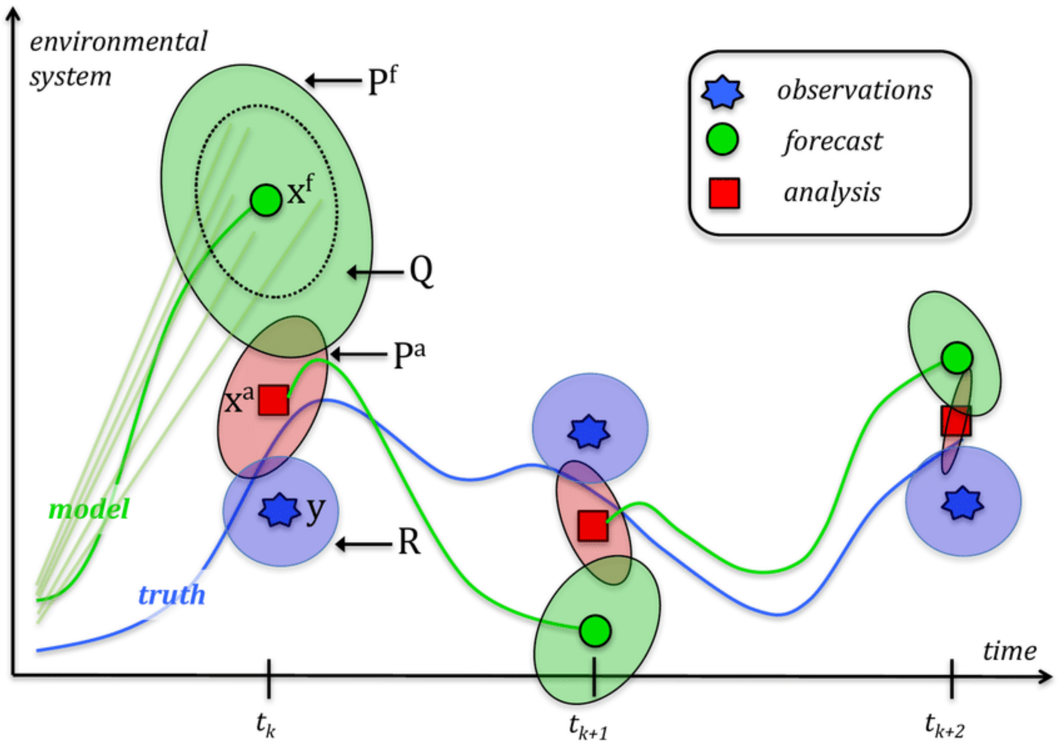




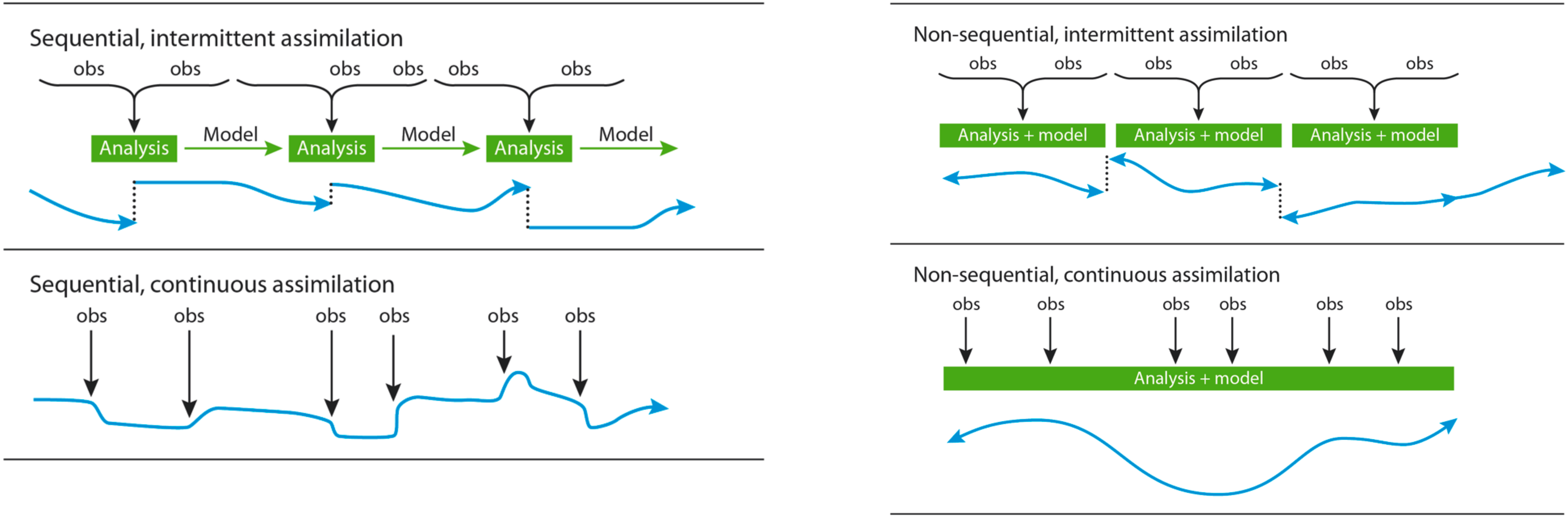
An operational model runs every day, restarting from the best available initial conditions, forced by predicted boundary conditions and producing forecasts.

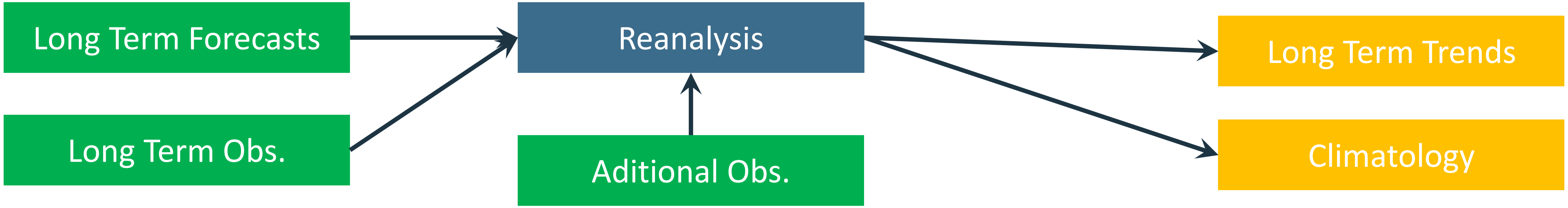
Data assimilation, analysis and reanalysis





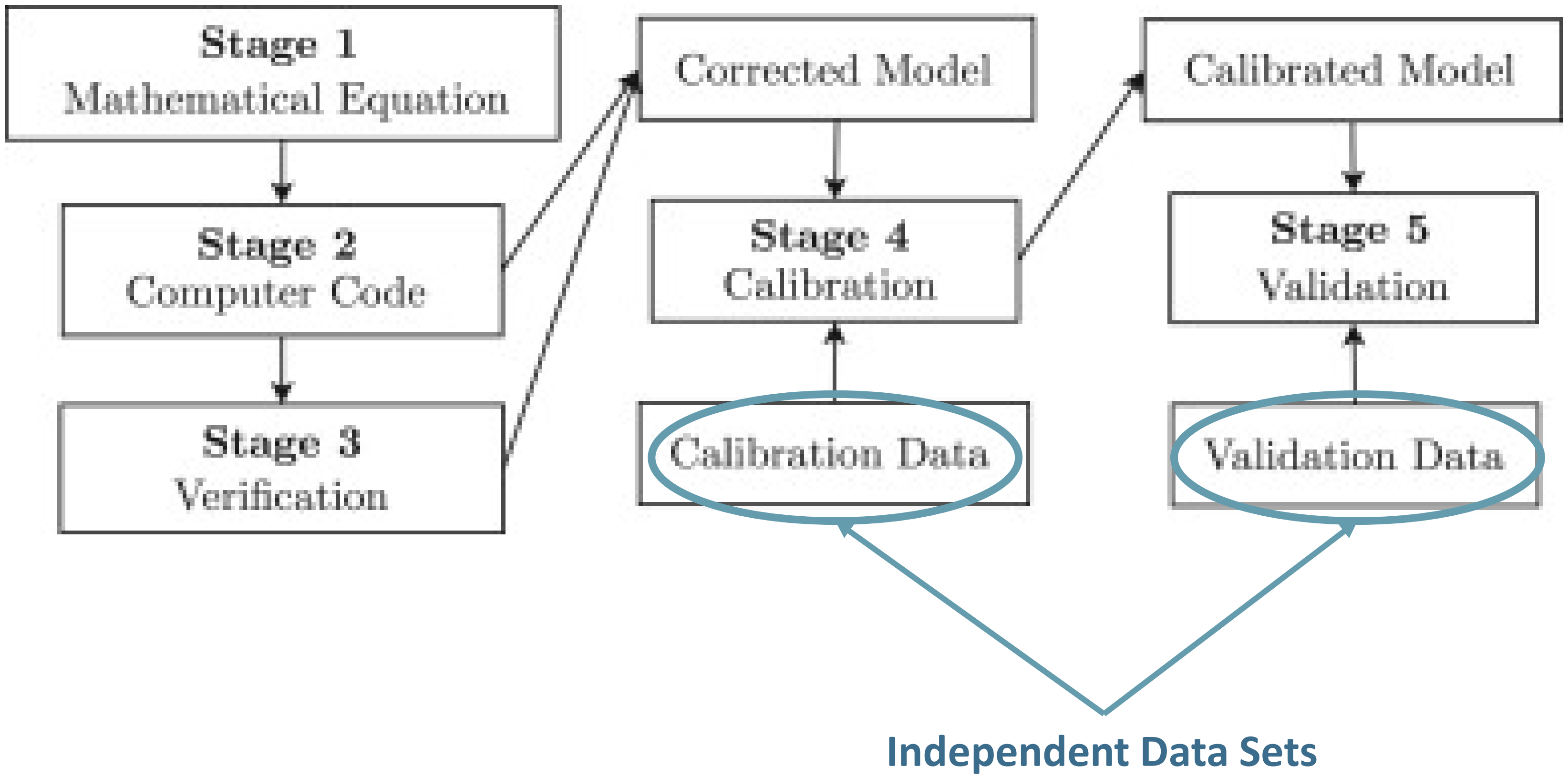
Data assimilation, analysis and reanalysis





Model Calibration & Validation

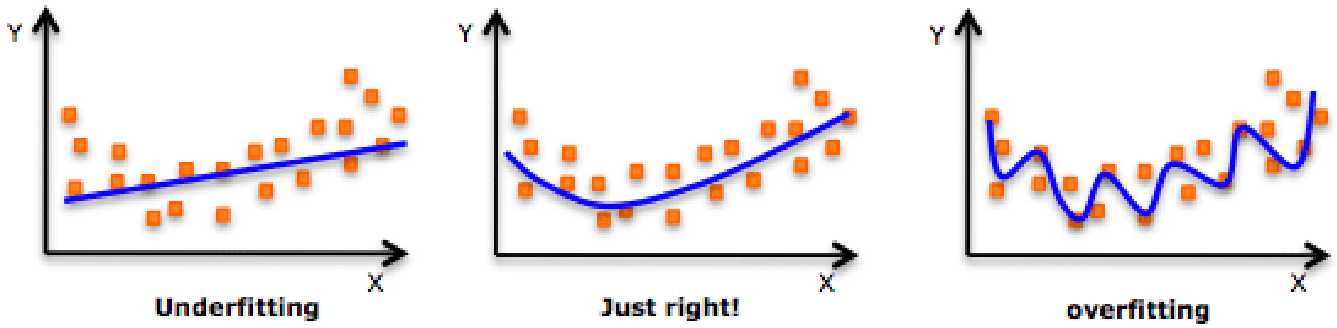
- Before we can use the model

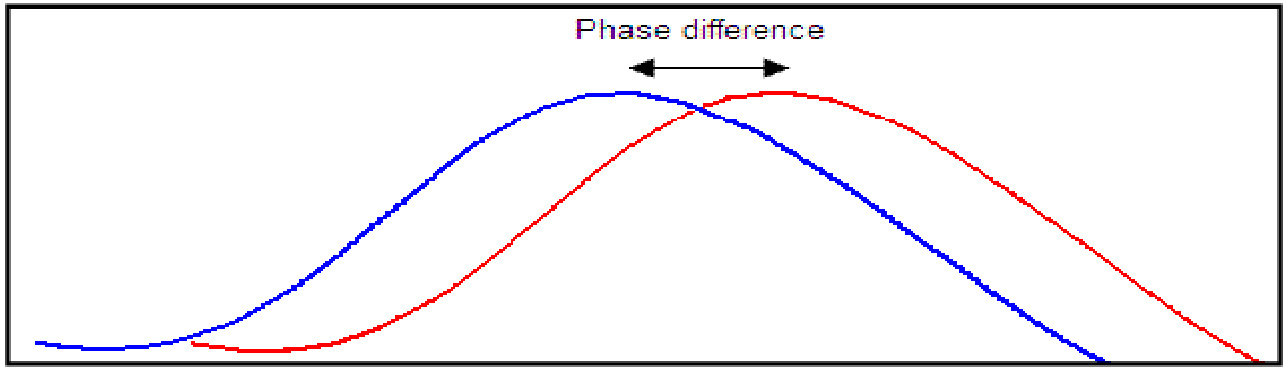


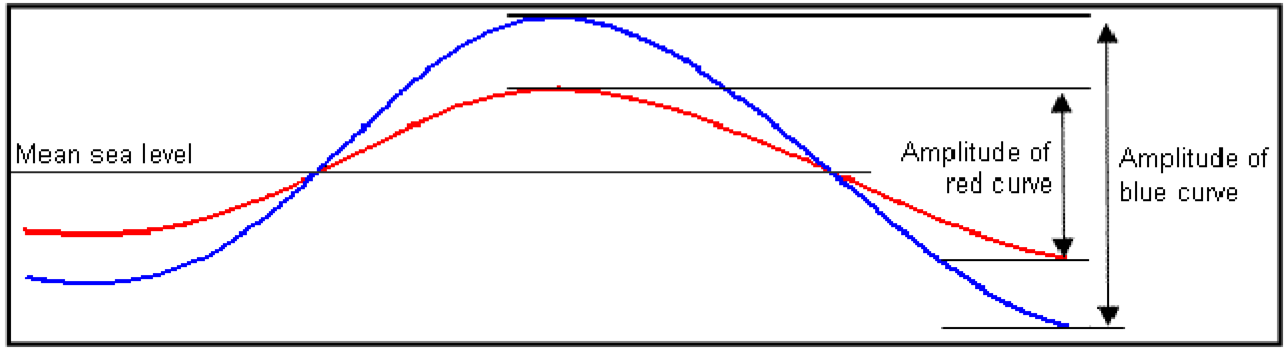
Model Calibration & Validation

- How to compare?

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N} (Predicted_i - Actual_i)^2}{N}}$$



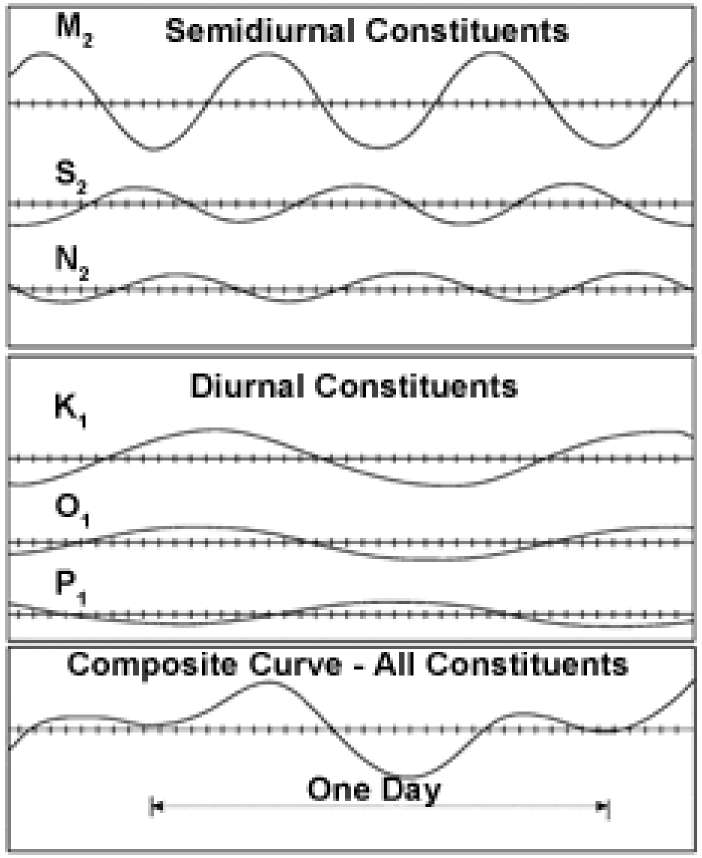


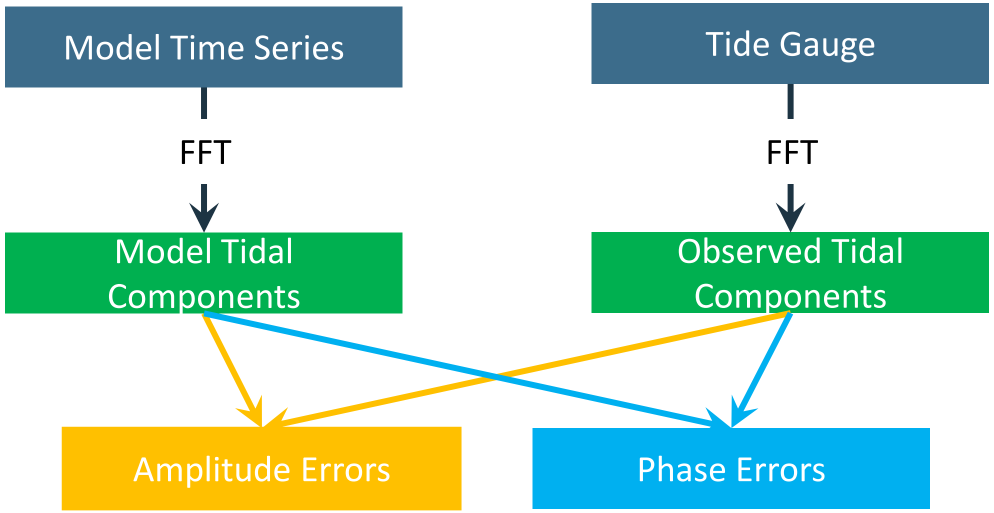


Model Calibration & Validation

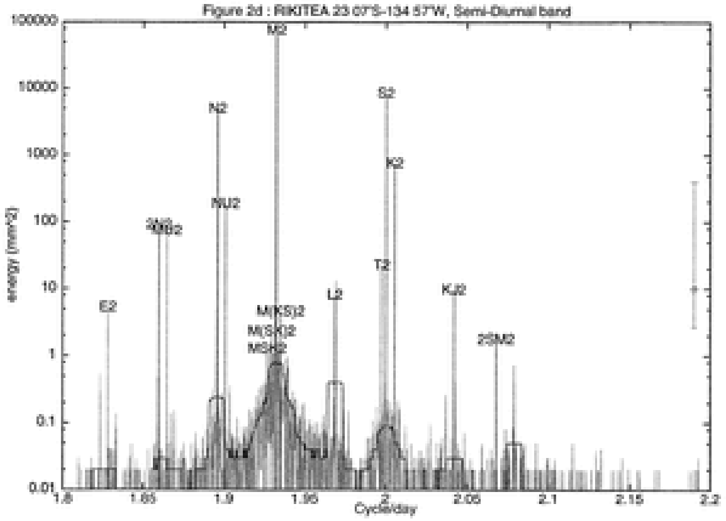
- Tidal Decomposition

$$H(t) = \sum_{i=1}^{i=N} A_i \cos(\omega_i t - \phi_i)$$





Power Spectrum



Conclusions

• What is a Model?	An approximate representation of Reality
• Transport of a property; Eulerian vs Lagrangian	Equivalent => Produce different model types
• Discretization in space and in time	Different mesh types / Explicit; Implicit; ADI
• Model limitations	Instability / Cr / Numerical diffusion
• Initial and boundary conditions	Necessary; Produce errors; Influence results
• Downscaling	Higher resolutions / Boundaries from GCM
• Types of models	Numerical method / Variables computed
• Operational modelling cycle	Model runs every day / restart from analysis
• Data assimilation, analysis and reanalysis	Combines model & Obs. producing analysis
• Model Calibration and Validation	Necessary; many ways of comparing data

Thank You !

Flávio Martins

fmartins@ualg.pt